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LS-173
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Radial and Tangential Winding Coil Probes for Sextupole Magnet Measurements

1. Introduction

Rotating coil probes of radial and tangential winding geometries for the measurements of the magnetic center, main field integral and multipole coefficients of sextupole magnets are described. Two sets of coils are sufficient for a probe of radial winding geometry. For a tangential winding probe, however, typically several sets of coils are required to measure the above magnetic parameters. The tangential coil geometry in this note is described with three sets of coils. The main sextupole field coefficients are defined as $b_2 = 1.0\text{cm}^{-2}$ and $a_2 = 0$ for the expression of the multipole field coefficients Eq. (1) in Reference 1.

2. Radial Winding Coils

The flux linkage for the two coils in Fig. 1, coil A and coil B, is given in Eqs. (5) and (6) in Reference 1 and is written here again,

$$\phi_{AB}(\theta) = LN_A \sum_{n=1}^{\infty} (C_n/R^{n-1})(r_{A2}{}^n/n)\ R_{fac}\ \cos n(\theta - \alpha_n) , \qquad (1)$$

$$R_{fac} = 1 - (-r_{A1}/r_{A2})^n - (N_B/N_A)\left[r_{B2}{}^n - (-r_{B1})^n\right]/r_{A2}{}^n , \qquad (2)$$

where N_A and N_B are the number of turns of coil A and coil B, respectively. From Eq. (2) the conditions for the rejection of the quadrupole and sextupole components for the measurements of higher multipole coefficients, $n \geq 4$, are given by

$$(r_{A2}{}^2 - r_{A1}{}^2) - (N_B/N_A)\ (r_{B2}{}^2 - r_{B1}{}^2) = 0 , \qquad (3)$$

$$\left[r_{A2}{}^3 - (-r_{A1})^3\right] - (N_B/N_A)\ \left[r_{B2}{}^3 - (-r_{B1})^3\right] = 0 . \qquad (4)$$

By introducing the following parameters

$$R = (r_{A2} - r_{A1})/2, \qquad r = (r_{B2} - r_{B1})/2,$$

$$C = (r_{A2} + r_{A1})/2, \qquad D = (r_{B2} + r_{B1})/2, \tag{5}$$

the above two conditions, Eqs. (3) and (4), are transformed into

$$RC = (N_B/N_A)\, rD,$$

$$(R^2 + 3C^2)/C = (r^2 + 3D^2)/D. \tag{6}$$

By solving Eq. (6), coil parameters for $N_B/N_A = 3.5$ as an example, are listed in Table 1.

When the cylinder rotation axis (CR) in Fig. 1 is displaced from the magnetic center by

$$Z_o = r_o \exp(i\theta_o), \tag{7}$$

it is shown in Eq. (10) Reference 1 that for a typical sextupole magnet, C_n $(n \neq 3)/C_3 << 1$; the quadrupole and sextupole components are given by

$$B'_\theta + iB'_r = (C_3/R^2)\ \exp(-i3\alpha_3)(Z_o + Z')^2. \tag{8}$$

Here $z' = r'\exp(i\theta')$ is a position on the surface of the rotating coil cylinder at $r' = r_{A2}$. The flux linkage for coil A is calculated from Eq. (8),

$$\phi'_A(\theta') = LN_A\ (C_3/R^2)\ \Big[r_o\ (r_{A2}{}^2 - r_{A1}{}^2)\ \cos(2\theta' + \theta_o - 3\alpha_3)$$

$$+ \frac{r_{A2}{}^3 - (-r_{A1})^3}{3} \cos 3(\theta' - \alpha_3) \Big]. \tag{9}$$

From Eq. (9) the $\cos 2\theta'$ term, which is the quadrupole field component due to the displacement of the magnetic center by Eq. (7), measures the magnetic center and the $\cos 3\theta'$ term measures the sextupole field integral. It should be noted that in Eq. (8) there is a dipole field component term which is proportional to r_o^2 and negligible. This is why the large sensitivity for n=1 in Table 1 is not important. This is also why finding the magnetic center for a sextupole magnet is defined for the minimization of the quadrupole field component instead of that of the dipole field component in this note.

When the conductor positions of r_{A2} and r_{A1}, and the coil plane have errors of Δr_A, as shown in Fig. 3 in Reference 1, Eq. (9) has additional terms. The corresponding expression with correction terms is

$$\phi'_A\ (\theta') = LN_A\ (C_3/R^2)(r_o Q + S), \tag{10}$$

with

$$Q=(r_{A2}^2 - r_{A1}^2)(1 + 2\frac{\Delta r_A}{r_{A2}})\cos(2\theta'+\theta_o-3\alpha_3)-3\Delta r_A \cdot r_{A1}\sin(2\theta'+\theta_o-3\alpha_3),$$

$$S=\frac{r_{A2}^3 - (-r_{A1})^3}{3}\ (1 + 3\frac{\Delta r_A}{r_{A2}})\cos 3(\theta'-\alpha_3) + \frac{3}{2}\ \Delta r_A \cdot r_{A1}^2 \sin 3(\theta'-\alpha_3),$$

where it is assumed that $r_{A2} = 2r_{A1}$, and the position of r_{A2} is the reference angular position. The correction terms are measured from the real and imaginary parts of Fourier analysis.

3. Tangential Winding Coil

Figure 2 shows the cross section of a tangential winding coil geometry with one Δ-coil and two $\pi/2$-coils. The two $\pi/2$-coils do have both roles for the $\pi/2$-coils and $\pi/3$-coils (see Reference 2) by series-connections of the two coils; one connection adds up the quadrupole field component and one connection adds up the main sextupole field components. The two series-connected coils will be called QQ and QS coils, and will have different numbers of turns of the coils, N_A and N_B respectively. Finally the three coils, Δ-coil, QQ-coil and QS-coil, are connected to have the flux linkage such that

$$\phi_\Delta(\theta) - \phi_{QQ}(\theta) - \phi_{QS}(\theta) = LN_\Delta \sum_{n=1}^{\infty} (C_n/R^{n-1}) \frac{r^n}{n} 2\sin n(\theta + \frac{\pi}{2} - \alpha_n) *$$

$$\left[\sin\left(\frac{n\Delta}{2}\right) - \{1 + (-1)^n\} \frac{N_{QQ}}{N_\Delta} \sin\left(\frac{n\pi}{4}\right) - \{1 - (-1)^n\} \frac{N_{QS}}{N_\Delta} \sin\left(\frac{n\pi}{4}\right)\right]. \tag{11}$$

From Eq. (11) the conditions for the rejection of the quadrupole and sextupole components are

$$C_2 \left[N_\Delta \sin(\Delta) - 2N_{QQ}\right] = 0, \tag{12}$$

$$C_3 \left[N_\Delta \sin(3\Delta/2) - 2N_{QS} \sin(3\pi/4)\right] = 0. \tag{13}$$

With a choice of the tangential coil parameters,

$$N_\Delta = 14,\ N_{QQ} = 2,\ N_{QS} = 4 \text{ and } \Delta = 15.3882^\circ,$$

relative sensitivities of the multipole coefficients are listed in Table 2.

For a sextupole magnet, $C_n\ (n \neq 3)/C_3 << 1$, when the CR is displaced from the magnetic center by Eq. (7), the flux linkages of QQ-coil and QS-coil are calculated from Eq. (8),

$$\phi'_{QQ}\ (\theta') = -LN_{QQ}\ (C_3/R^2)\ 4r_o r^2 \sin(2\theta' + \theta_o - 3\alpha_3), \tag{14}$$

$$\phi'_{QS}\ (\theta) = -LN_{QS}\ (C_3/R^2)\ (2\sqrt{2}\ r^3/3) \cos 3(\theta' - \alpha_3). \tag{15}$$

The sextupole field integral is measured from Eq. (15) which does not depend on the position of the location of the cylinder axis. After finding the sextupole strength, the displacement of the CR from the magnetic center is found from Eq. (14).

4. References

1. A Radial Coil Probe for Quadrupole Magnet Measurement, S. H. Kim, LS-166.

2. Tangential Winding Coil Probes for Dipole, Quadrupole and Sextupole Magnet Measurements, S. H. Kim, LS-167.

Table 1. Radial winding coil parameters for sextupole magnet measurements. Bar chart shows the relative sensitivity. $r_{A2}=4$, $r_{A1}=2$, $r_{B2}=2.50656$, $r_{B1}=1.68946$, $N_A=6$, $N_B=21$.

	A	B	C
1	Radial coil probe for sextupole magnet measurements		
2			
3	ra2	4	
4	ra1	2	
5	rb2	2.50656	
6	rb1	1.68946	
7	na	6	
8	nb	21	
9	case no	2	
10			
11			
12	n	rfac	
13	1	-2.1715175	
14	2	7.62688E-07	
15	3	5.04186E-05	
16	4	0.509198023	
17	5	0.646017304	
18	6	0.792322563	
19	7	0.866621141	
20	8	0.916421212	
21	9	0.948308787	
22	10	0.966978246	
23	11	0.97974416	
24	12	0.987036927	
25	13	0.992033547	
26	14	0.994920352	

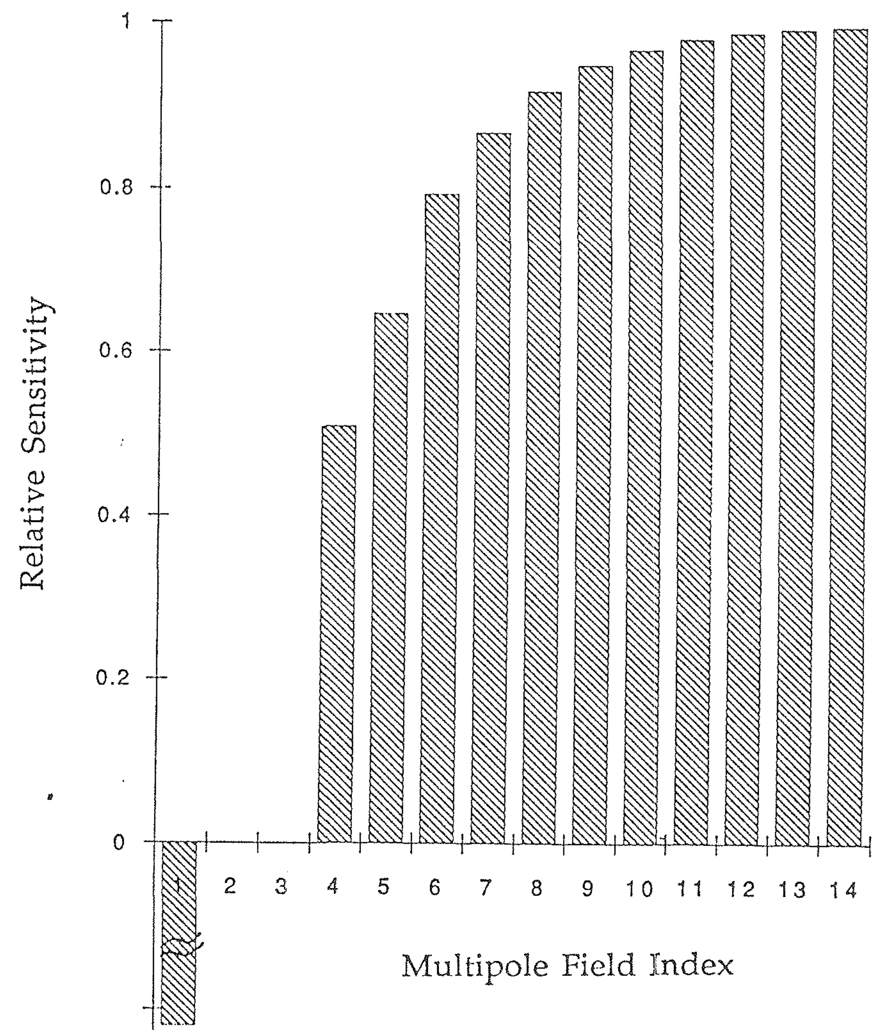
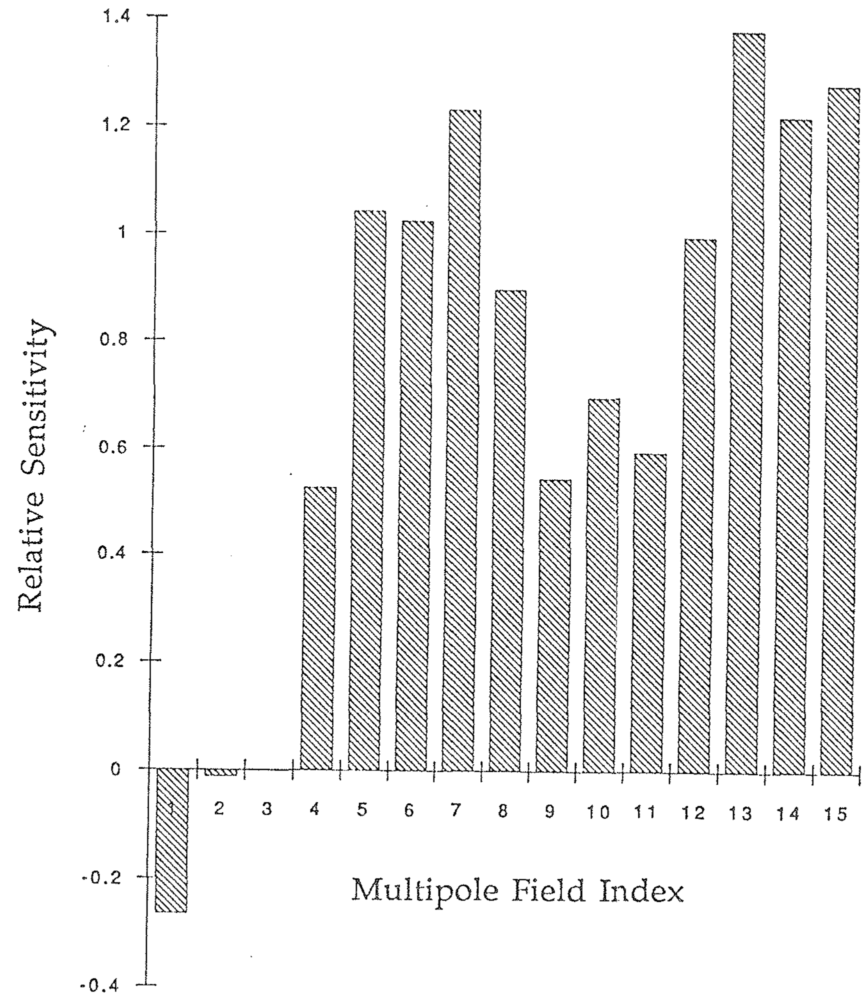


Table 2. Tangential winding coil parameters for sextupole magnet measurements. N_{Δ}=14, N_{QQ}=2, N_{QS}=4, Δ=15.3882°.

	A	B	C
1	Tangential coil probe for sextupole magnet measurements		
2			
3	ndelta	14	
4	nqq	2	
5	nqs	4	
6	delta	15.3882	
7	pi	3.14159265	
8	case no	4	
9			
10	n	sensitivity	
11	1	-0.270176879	
12	2	-0.020356728	
13	3	-0.012008076	
14	4	0.511689016	
15	5	1.026172626	
16	6	1.007046737	
17	7	1.211625989	
18	8	0.879256459	
19	9	0.531055024	
20	10	0.68842362	
21	11	0.591558408	
22	12	0.99917381	
23	13	1.388798069	
24	14	1.238283393	
25	15	1.307310206	



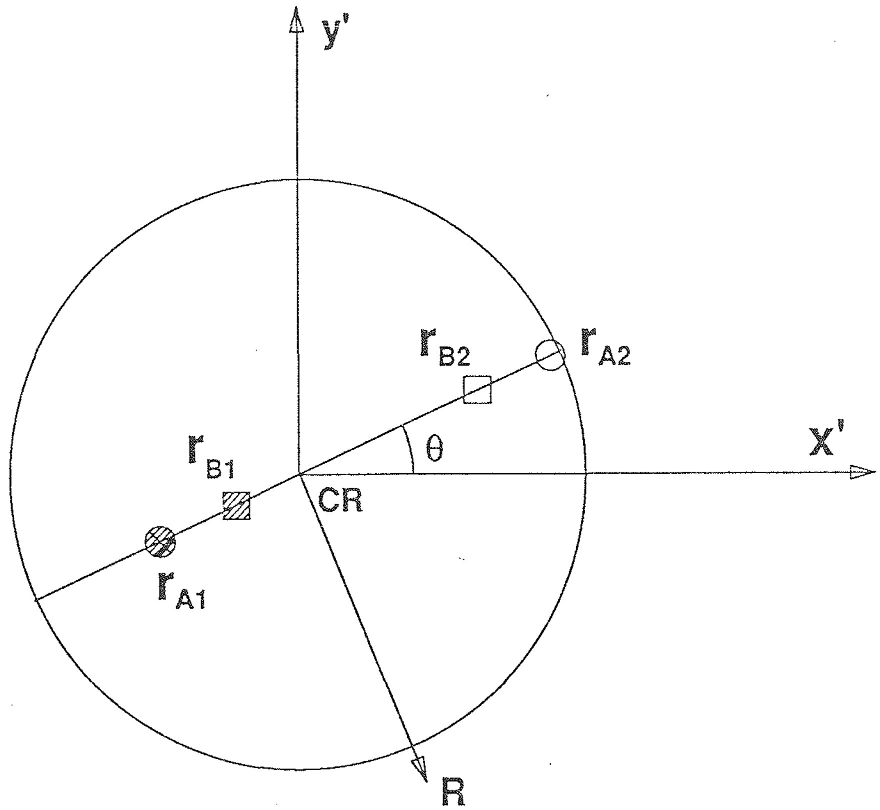


Fig. 1. Cross section of two sets of radial winding coils in a cylinder. Coil A is located at (r_{A2}, θ) and (r_{A1},θ+π), and has a number of turns N_A. Coil B is located at (r_{B2},θ) and (r_{B1},θ+π) with N_B turns.The cylinder rotation axis (CR) is located at the origin of the x'y'-coordinate system. The CR is in the plane of the two coils. The radius of the measuring magnet aperture can be chosen as the reference radius R. The reference angular position, θ = 0, is the direction of the CR/r_{A2}.

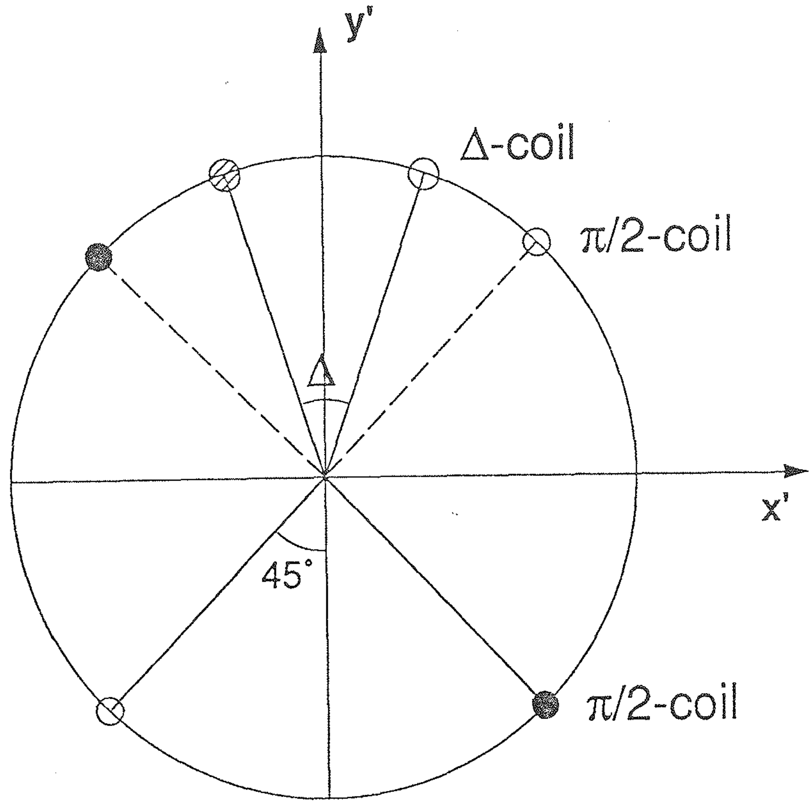


Fig.2. Cross section of a tangential winding coil for sextupole magnet measurements. It consists of one Δ-coil and two π/2-coils which are used for the rejection of the quadrupole field for the measurements of the magnetic center, and for the rejection of the sextupole field for harmonic analysis. The two π/2-coils, one in the +y-plane and one in the -y-plane, are connected as shown in the second term of Eq. (11). The two π/2-coils are also used for the measurements of the sextupole field integral by connecting them as shown in the third term of Eq. (11). The x′-axis is the reference angular position, $\theta = 0$.